Analysis of innovation based on financial structure

You-hua Chen^a, Pu-yan Nie^b and Xiao-wei Wen^{c*}

^aCollege of Economics & Management and Guangdong Center for Rural Economic Studies, South China Agricultural University, Guangzhou, 510642, People's Republic of China; ^bInstitute of Industrial Economics, Jinan University, Guangzhou, 510632, People's Republic of China; ^cCollege of Economics & Management, South China Agricultural University, Guangzhou, 510642, People's Republic of China

(Received 21 April 2015; accepted 24 August 2015)

This paper examines the interaction between innovation and financial structure under monopoly. We characterise the effects of debt levels on innovative investment by considering a limited liability effect. On one hand, higher debt levels promote both innovative investment and the outputs. On the other hand, shareholders' net benefits are reduced by higher debt levels and net profit per debt is correspondingly reduced by higher debt level under positive net profit. More importantly, this study captures the interaction between financial structure and industrial organisation without restriction of the interior point.

Keywords: financial structure; innovation; monopoly; corporate finance; debt levels; game theory

JEL classification: C70, D42, L1, G0

1. Introduction

As we known, financial structures have extremely important effects on firms' strategies. In their pioneering work, Brander and Lewis (1986) established the significant connection between financial structure and industrial organisation and many authors further developed this relationship. Two years later, Maskimovic (1988) explored this relation in dynamics and derived some interesting conclusions. In addition, Aybar-Arias, Casino-Martinez, and Lopez-Gracia (2012) considered the optimal financial structure. Meanwhile, Cull, Demirgüç-Kunt, and Lin (2013) discussed the effects of financial structures on the economic development while Uras (2014) examined the relationship between the financial structures and the total factor productivity.

A firm's profit closely relates to its finance and financial structure (Nie, 2011; Riordan, 2003; Showalter, 2010). Chevalier (1995) empirically confirmed the existence of interactions between capital markets and product markets. Riordan (2003) summarised the literature related to the relationship between capital and product markets at the nexus of industrial organisation and corporate finance. According to Riordan's view, on one hand, capital market constraints on an individual firm are mainly determined by the level of the industry and critically depend on product market competition. On the other hand, capital markets constrain the product strategy of firms and thereby influence product market performance. Since firms' behaviours and strategies closely relate to

*Corresponding author. Email: wxwcn126@163.com

This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

^{© 2015} The Author(s). Published by Taylor & Francis.

their financial states, capital has deep effects on firms or industries. Furthermore, many industries are capital-intensive and capital is decisive in the situation of these firms (Ramalho & da Silva, 2009). From this aspect, how to obtain enough capital is a vital topic in management issues. Riordan (2003) stressed that capital structure heavily affects earnings of firms, which can also be seen in Harris and Raviv (1991). Undoubtedly, this is an exceedingly interesting topic both in industrial organisation and in finance, so this work is focused on capital structure in industrial economics.

Harris and Raviv (1991) pointed out that models of capital structure employing features of industrial organisation theory entail two categories. One addresses the relationship between firms' capital structure and their strategies in a competing product market. The other focuses on firms' capital structures and the characteristics of their products or inputs.

The first category follows the classic papers of Brander and Lewis (1986) and Maskimovic (1986), and the basic thought of Jansen and Meckling (1976) was employed to capture the relationship between firms' capital structure and their strategies. Maskimovic (1988) investigated this relationship in a dynamic environment. Brander and Lewis (1988) further addressed bankruptcy cost under oligopoly structure. Tarzijan (2007) recently exploited the effect of capital structure to deter invaders if there are multiple incumbents.

The other category combined industrial organisation approaches with capital structure to identify product market. In this way, Titman (1984) initially found that liquidation of a firm may impose costs on its customers. These incurred costs were transferred to shareholders through the lower prices of firms' products. Harris and Raviv (1991) introduced this in their significant survey about capital structure theory.

This work falls into the first category and aims to capture the relationship between capital structure and innovative investment under monopoly. In addition, this article characterises the effects of debt levels on innovative investment for a monopolist. For the innovations, this paper follows the interesting papers of Vives (2008) and Sacco and Schmutzler (2010).

In theory, Brander and Lewis' (1986) significant work motivates this idea to address the capital structure and innovative investment strategy. In applications, underinvestment in innovation is very popular, which stimulates further research on this topic to trace the essence. This paper discusses the monopolisation case.

The rest of this paper is organised as follows: the model is established in Section 2, and analysed in Section 3, where the effects of capital structure on shareholder value and debt value are characterised. Then some remarks and conclusions are presented in the final section.

2. Model set up

للاستشارات

Model of a monopolist in debt with limited liability effect is established in this section. The monopolist produces product q and the corresponding price of this product is p.

Consumers. Given a constant A > 0, the utility of consumers is

$$u(p,q) = Aq - \frac{1}{2}q^2 - pq.$$
 (1)

The demand is induced by equation (1), which is stated as follows

$$q = A - p. \tag{2}$$

In general, A > 0 is large enough such that demand is large enough.

Monopolist. Given innovative investment I, the operating profit of the monopolist, without considering debt, is

$$\pi(q, z, I) = pq - c(I)q + g(z, q) - \frac{1}{2}I^2,$$
(3)

where z is a random variable that represents the effects of an uncertain environment on the fortunes of the monopolist. The random variable z uniformly distributes over the interval $[\underline{z}, \overline{z}]$ associated with density function $f(z) = \frac{1}{\overline{z}-\overline{z}}$. g(z, q), a real function, indicates the joint effects of z and q. c(I) is the marginal cost incurred by production. $\frac{1}{2}I^2$ stands for the investment cost.¹ The above operating profit meets the relations $\frac{\partial \pi}{\partial z} > 0$, $\frac{\partial^2 \pi}{\partial q \partial z^2} = 0$ and $\frac{\partial^2 \pi}{\partial q^2} < 0$. $\frac{\partial \pi}{\partial z} > 0$ indicates that higher value of z should yield higher operating profit. $\frac{\partial^2 \pi}{\partial q^2} < 0$ means that the operating profit function is concave, which guarantees the existence of a unique solution for equation (3). Actually, the random variable z and the quantity may jointly have positive or the negative effects on firms' profits. In the other case, the random variable and the quantity do not interact with the profits of firms. Therefore, this study entails three cases:

$$g(z,q) = \begin{cases} zq \\ -zq \\ z \end{cases}$$
 We further note that Brander and Lewis (1986) considered $\frac{\partial^2 \pi}{\partial q \partial z} > 0$
and $\frac{\partial^2 \pi}{\partial z}$ O Eurthermore, they pointed out that $\frac{\partial^2 \pi}{\partial z} < 0$ is more but it is possible. Since

and $\frac{\partial^2 \pi}{\partial q \partial z} = 0$. Furthermore, they pointed out that $\frac{\partial^2 \pi}{\partial q \partial z} < 0$ is rare, but it is possible. Since we have $\frac{\partial^2 \pi}{\partial q \partial z} > 0$, $\frac{\partial^2 \pi}{\partial q \partial z} < 0$ and $\frac{\partial^2 \pi}{\partial q \partial z} = 0$ in three cases, this paper addresses three cases to capture this problem.

Shareholders. Assuming that the monopolistic producer should carry out debt financing to maintain its business. Given predetermined debt level D, the value that goes to the shareholders after debt financing and the production decision is the equity value, which is represented by V. After production occurs, by virtue of financial policies, the monopolist is obliged to pay creditors D out of current profits. The firm sets its quantity and innovative investment to maximise the expected value of the monopolist to the shareholders. The value to shareholders and the corresponding notations are all similar to that of Brander and Lewis (1986).

$$V(q,I) = \int_{\hat{z}}^{\hat{z}} [\pi(q,z,I) - D] f(z) dz$$
(4)

where \hat{z} is completely determined by

$$\pi(q, \hat{z}, I) - D = 0.$$
(5)

We further stipulate that $\underline{z} < \hat{z} < \overline{z}$, which is similar to that of Brander and Lewis (1986). When $z = \hat{z}$, the monopolist's operating profit can just cover its debt obligations without anything left over. If $z < \hat{z}$, the monopolist pays all its earnings to debt holders and earns zero, and this state seems seriously bad. $z > \hat{z}$ indicates positive profits for the monopolist. V/D represents the net profit per debt.

From equation (5) and the envelope theorem, we obtain the following Proposition.

Proposition 1. Higher debt levels decrease shareholder value. Net profit per debt is also reduced with higher debt level.

Proof. From equation (4), we obtain $\frac{\partial V(q,I)}{\partial D} = \frac{\partial \int_{\hat{z}}^{\bar{z}} [\pi(q,z,I)-D]f(z)dz}{\partial D} < 0$ for $\hat{z} < z < \bar{z}$ and $\bar{z} < z$ by employing the envelope theorem. For the same reason, we have $\frac{\partial [V(q,I)/D]}{\partial D} < 0$. Net profit per debt becomes less with higher debt level.



Conclusions are achieved and the proof is therefore complete.

Remarks: Proposition 1 illustrates that higher debts reduce stockholder value. In other words, capital structure has significant effects on firms' and shareholders' value. The result of Proposition 1 is different from the capital structure theory of Modigliani and Miller (1958).

The model is given by equations (1)–(5). The following assumption is launched, which is similar to that in Vives (2008).

Assumption. c(I) is convex and c'(I) < 0.

c'(I) < 0 indicates that innovative investment efficiently reduces the incurred production cost. This hypothesis is extremely rational and very moderate, and also appeared in other innovation theory papers, such as Sacco and Schmutzler (2010). To make our study more focused, other factors, such as bankruptcy costs and tax advantages of debt, are all not discussed in the paper, although these factors are all considerably important.

3. Model analyses

First, we show the existence and uniqueness of the solution to maximise the value of shareholders. Equation (4) is restated as follows.

$$V(q,I) = \int_{\hat{z}}^{\hat{z}} \left[(A-q)q - c(I)q + g(z,q) - \frac{1}{2}I^2 - D \right] f(z)dz$$
(6)

with $g(\hat{z},q) = D - [(A - q)q - c(I)q - \frac{1}{2}I^2].$

Although equation (6) is not necessary concave, we have the following conclusions.

Proposition 2. Equation (4) has a unique solution.

Proof. See the Appendix.

Remarks. In general, it is exceedingly difficult to obtain the concavity of V from the concavity of π . Brander and Lewis (1986) stipulated the solution to be the strictly interior point, which simplifies the model to a great degree and guarantees the concavity of equation (6). This article discusses general cases or no restriction of the interior point solution is made. The concavity is therefore not guaranteed but the unique solution is shown.

This article correspondingly captures this industry in three cases according to three types of formulations of g(z, q).

3.1. g(z, q) = z

If g(z, q) = z and \hat{z} , $(A - q)q - c(I)q - \frac{1}{2}I^2 - D$ is concave in q and I. The solution is determined by

$$A - c(I) - 2q = 0$$
 and $-c'(I)q - I = 0$ (7)

If $\underline{z} \leq \hat{z} < \overline{z}$, equation (6) is rewritten as the following formulation, which is also equation (A1).

$$V(q,I) = \frac{1}{2(\bar{z}-\underline{z})} \left\{ \bar{z} + \left[(A-q)q - c(I)q - \frac{1}{2}I^2 - D \right] \right\}^2$$
(8)

Proposition 2 indicates that the solution of the above system is determined by the first-order optimal conditions of equation (8).



$$\frac{\partial V(q,I)}{\partial q} = -\frac{A - c(I) - 2q}{\overline{z} - \underline{z}} \left[(A - q)q - c(I)q - \frac{1}{2}I^2 - D + \overline{z} \right] = 0$$
(9)

$$\frac{\partial V(q,I)}{\partial I} = -\frac{-c'(I)q - I}{\bar{z} - \underline{z}} \left[(A - q)q - c(I)q - \frac{1}{2}I^2 - D + \overline{z} \right] = 0 \tag{10}$$

Let the solution of equation (11) be (q^*, I^*) . Then, $\underline{z} \le D - [(A - q^*)q^* - c(I^*)q^* - \frac{1}{2}(I^*)^2] < \overline{z}$ or

$$0 < \bar{z} - D + \left[(A - q^*)q^* - c(I^*)q^* - \frac{1}{2}(I^*)^2 \right]$$
(11)

If $\underline{z} \le \hat{z} < \overline{z}$, the solution is at the corner and the solution is also determined by equation (7) and

$$\hat{z} - D + \left[(A - q^*)q^* - c(I^*)q^* - \frac{1}{2}(I^*)^2 \right] = 0$$
(12)

For g(z, q) = z, the following result holds:²

Proposition 3. Innovative investment and product quantity have no relation with debt levels.

Proof. Equation (7) implies that innovative investment and product quantity have no relation with debt levels. Conclusions are achieved and the proof is therefore complete. \Box

Remarks. This is consistent with the conclusions in Brander and Lewis (1986), while innovative investment is highlighted in this study.

3.2. g(z, q) = zq

Here we address g(z, q) = zq. If $\hat{z} < \underline{z}$, conclusions are the same as Proposition 3. If $\underline{z} \le \hat{z} < \overline{z}$, we have the following formulation, which is also equation (A2),

$$V(q,I) = \frac{1}{2(\bar{z}-\underline{z})} \left\{ q^{\frac{1}{2}} + q^{-\frac{1}{2}} \left[(A-q)q - c(I)q - \frac{1}{2}I^2 - D \right] \right\}^2.$$
 (13)

In equation (13), we can easily examine that $q^{\frac{1}{2}}\overline{z} + q^{-\frac{1}{2}}[(A-q)q - c(I)q - \frac{1}{2}I^2 - D]$ is concave. Moreover, $\underline{z} \leq \hat{z} < \overline{z}$ implies $q^{\frac{1}{2}}\overline{z} + q^{-\frac{1}{2}}[(A-q)q - c(I)q - \frac{1}{2}I^2 - D > 0$. The solution to equation (13) is determined by

$$f_{1} = \frac{\partial \{q^{\frac{1}{2}}\overline{z} + q^{-\frac{1}{2}}[(A-q)q - c(I)q - \frac{1}{2}I^{2} - D]\}}{\partial q}$$

= $\frac{1}{2}q^{-\frac{1}{2}}\overline{z} + \frac{1}{2}Aq^{-\frac{1}{2}} - \frac{3}{2}q^{\frac{1}{2}} - \frac{1}{2}c(I)q^{-\frac{1}{2}} + \frac{1}{4}q^{-\frac{3}{2}}I^{2} + \frac{1}{2}q^{-\frac{3}{2}}D = 0$ (14)

$$f_2 = \frac{\partial \left\{ q^{\frac{1}{2}\overline{z}} + q^{-\frac{1}{2}} \left[(A-q)q - c(I)q - \frac{1}{2}I^2 - D \right] \right\}}{\partial I} = q^{-\frac{1}{2}} \left[-\frac{dc(I)}{dI}q - I \right] = 0.$$
(15)

For g(z, q) = zq, equation (14) yields the following conclusions.

Proposition 4. If g(z, q) = zq and the solution lies at the corner, we have $\frac{\partial q}{\partial D} > 0$ and $\frac{\partial I}{\partial D} > 0$.



Proof. From equation (14), we have $\frac{\partial f_1}{\partial q} < 0$, $\frac{\partial f_1}{\partial D} > 0$, $\frac{\partial f_2}{\partial D} > 0$ and $\frac{\partial f_2}{\partial q} > 0$. Implicit function theorem indicates $\frac{\partial q}{\partial D} > 0$ and $\frac{\partial I}{\partial D} > 0$. These conclusions are achieved and the proof is therefore complete.

Remarks. $\frac{\partial q}{\partial D} > 0$ indicates that higher debt levels bring about more quantity of products. This is consistent with the conclusions of the extant literature (Brander & Lewis, 1986; Dixit, 1980). Dixit (1980) argued that higher debt level acts as an important commitment of more outputs. Higher debt level correspondingly improves innovative investment according to $\frac{\partial I}{\partial D} > 0$.

Without the hypothesis of the interior point solution, the conclusion for product quantity is the same as that of Brander and Lewis (1986). When innovative investment is introduced, this study shows that higher debt level also promotes innovative investment, which is consistent with many social phenomena.

Here we address the relation between debt levels and shareholder value if g(z, q) = zq. According to equation (13), by envelope theorem we have the following relation.

3.3. g(z, q) = -zq

Here we discuss g(z, q) = -zq. If $\hat{z} < \underline{z}$, conclusions are the same as Proposition 2. If $\underline{z} \le \hat{z} < \overline{z}$, the following equation, which is also equation (A3), holds.³

$$V(q,I) = \frac{-1}{2(\bar{z}-\underline{z})} \left\{ q^{\frac{1}{2}} - q^{-\frac{1}{2}} \left[(A-q)q - c(I)q - \frac{1}{2}I^2 - D \right] \right\}^2$$
(16)

Equation (16) is concave. The optimal product quantity and innovative investment are determined by the following formulation.

$$\frac{\partial V(q,I)}{\partial q} = -\frac{\frac{1}{2}q^{-\frac{1}{2}}\bar{z} - \frac{1}{2}Aq^{-\frac{1}{2}} + \frac{3}{2}q^{\frac{1}{2}} + \frac{1}{2}c(I)q^{-\frac{1}{2}} - \frac{1}{4}q^{-\frac{3}{2}}I^2 - \frac{1}{2}q^{-\frac{3}{2}}D}{\bar{z} - \bar{z}}$$

$$\left\{q^{\frac{1}{2}}\bar{z} - q^{-\frac{1}{2}}\left[(A-q)q - c(I)q - \frac{1}{2}I^2 - D\right]\right\} = 0$$
(17)

$$\frac{\partial V(q,I)}{\partial I} = -\frac{-c'(I)q - I}{\bar{z} - \underline{z}} \left\{ q^{\frac{1}{2}} \overline{z} - q^{-\frac{1}{2}} \left[(A - q)q - c(I)q - \frac{1}{2}I^2 - D \right] \right\} = 0$$
(18)

Actually, $q^{\frac{1}{2}}\bar{z} - q^{-\frac{1}{2}}[(A-q)q - c(I)q - \frac{1}{2}I^2 - D] > 0$ by virtue of $\underline{z} \le \hat{z} < \bar{z}$. Therefore, equations (17) and (18) imply that the equilibrium state is determined by the following first-order optimal conditions

$$f_{3} = -\left[\frac{1}{2}q^{-\frac{1}{2}}\overline{z} - \frac{1}{2}Aq^{-\frac{1}{2}} + \frac{3}{2}q^{\frac{1}{2}} + \frac{1}{2}c(I)q^{-\frac{1}{2}} - \frac{1}{4}q^{-\frac{3}{2}}I^{2} - \frac{1}{2}q^{-\frac{3}{2}}D\right] = 0,$$
(19)

$$f_4 = -c'(I)q - I = 0 (20)$$

Similar to Section 3.2, we have the following conclusions.

Proposition 5. If g(z, q) = -zq and the solution lies at the corner, we have $\frac{\partial q}{\partial D} > 0$ and $\frac{\partial I}{\partial D} > 0$.

Proof. From equations (19) and (20), we have $\frac{\partial f_3}{\partial q} < 0$, $\frac{\partial f_3}{\partial I} > 0$, $\frac{\partial f_4}{\partial I} < 0$ and $\frac{\partial f_3}{\partial q} > 0$. The implicit function theorem indicates $\frac{\partial q}{\partial D} > 0$ and $\frac{\partial I}{\partial D} > 0$. The conclusions are achieved and the proof is therefore complete.



The relationship between financial structure and innovative innovation is characterised and the optimal debt levels are discussed in this paper. Without the restriction of the interior point solution, it seems more difficult to handle this. In three cases, shareholders' net profit is reduced with higher debt levels. For g(z, q) = zq and g(z, q) = -zq, higher debt levels yield higher product quantity and higher innovative investment. For g(z, q) = z, debt levels have no effect on product quantity and innovative investment.

Combining Propositions 1, 3 and 4, we learn that firm debt has two opposite effects because higher debts motivate firms' innovative investment and outputs while decreasing shareholders value. In other words, managers should take both the value decrease effect (higher debts decrease shareholders' value) and the competitive strategy effects (higher debts increase innovation and outputs) into consideration when they make debt financing decisions.

4. Concluding remarks

This paper characterises the relation between financial structure and innovative investment under monopoly. In addition, this study characterises the relationship between debt levels and innovative investment, along with the quantity of products. Interestingly, for g(z, q) = zq and g(z, q) = -zq, although higher debt levels lower stockholders' value, they cause both higher innovative investment and higher quantity of products. And the motivated effects of debt on innovative investment and quantity sustain the incentive debt theory and are a key issue of capital structure-industrial organisation theory. Net profit per debt is reduced with higher debt level under positive net profits. Furthermore, this paper employs a linear demand function, which is tractable, and it is easy to extend to general cases.

Compared with Brander and Lewis (1986), this paper addresses the monopolisation innovation with limited liability effects. More importantly, this study discusses the solution at the corner, which seems more practical and more complicated and this study illustrates that debt has no effect on output and innovative investment under some circumstances, such as g(z, q) = z.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work is partially supported by the Foundation for High-level Talents in Higher Education of Guangdong, GDUPS (2012); The National Natural Science Foundation of PRC (71271100, 71401057); The Guangdong Social Science Foundation (GD13YLJ02); The Fundamental Research Funds for the Central Universities, Characteristic and Innovative Foundation (Humanities and Social Sciences) for Higher Education of Guangdong (2014); Research Centre for the Industrial Development of Guangdong and its Regional Cooperation with Hong Kong, Macau and Taiwan; The Soft Science Project of Guangdong Province (2014A070704008); The China Scholarship Council (201508440104).

Notes

1. This seems like that in Sacco and Schmutzler (2010).



- 2. Furthermore, if $D [(A q^*)q^* c(I^*)q^* \frac{1}{2}(I^*)^2] > \overline{z}$, the debt holders do not enter into this industry. This case is not discussed. In other situations, we will not discuss $\hat{z} \ge \overline{z}$.
- 3. Brander and Lewis (1986) pointed out that $\frac{\partial^2 \pi}{\partial q \partial z} < 0$ is rare .In this case, from equation (16) we have $V \le 0$.

ORCID

You-hua Chen D http://orcid.org/0000-0001-5697-9295

References

- Aybar-Arias, C., Casino-Martinez, A., & Lopez-Gracia, J. (2012). On the adjustment speed of SMEs to their optimal capital structure. *Small Business Economics*, 39, 977–996.
- Brander, J. A., & Lewis, T. R. (1986). Oligopoly and financial structure: The limited liability effect. *American Economic Review*, 76, 956–970.
- Brander, J. A., & Lewis, T. R. (1988). Bankruptcy costs and the theory of oligopoly. *Canadian Journal of Economics*, 21, 221–243.
- Chevalier, J. A. (1995). Capital structure and product-market competition: Empirical evidence from the supermarket industry. *American Economic Review*, 85, 415–435.
- Cull, R., Demirgüç-Kunt, A., & Lin, J. Y. (2013). Financial structure and economic development: A reassessment. *The World Bank Economic Review*, 27, 470–475.
- Dixit, A. (1980). The role of investment in entry deterrence. Economic Journal, 90, 95-106.
- Harris, M., & Raviv, A. (1991). The theory of capital structure. *The Journal of Finance, 46*, 297–355.
- Jansen, M. C., & Meckling, W. (1976). Theory of the firm: Managerial behavior, agency costs, and capital structure. *Journal of Financial Economics*, 3, 305–360.
- Maskimovic, V. (1986). *Optimal capital structure in oligopolies*, Doctoral dissertation, Havard University.
- Maskimovic, V. (1988). Capital structure in repeated oligopolies. Rand Journal of Economics, 19, 389–407.
- Modigliani, F., & Miller, M. H. (1958). The cost of capital, corporation finance and the theory of investment. *The American Economic Review*, 48, 261–297.
- Nie, P. Y. (2011). Spatial maintenance goods under monopoly. Ekonomska Istrazivanja-Economic Research, 24, 16–26.
- Ramalho, J. J. S., & da Silva, J. V. (2009). A two-part fractional regression model for the financial leverage decisions of micro, small, medium and large firms. *Quantitative Finance*, 9, 621–636.
- Riordan, M. H. (2003). How do capital markets influence product market competition? *Review of Industrial Organization*, 23, 179–191.
- Sacco, D., & Schmutzler, A. (2010). Is there a U-shaped relation between competition and investment. *International Journal of Industrial Organization*, 28, 65–73.
- Showalter, D. (2010). Entry deterrence through credit denial. International Review of Economics & Finance, 19, 539–554.
- Tarzijan, J. (2007). Capital structure and entry deterrence with multiple incumbents. *The B.E. Journal of Economic Analysis & Policy*, 7(1), 1–18.
- Titman, S. (1984). The effect of capital structure on a firm's liquidation decision. *Journal of Financial Economics*, 13, 137–151.
- Uras, B. R. (2014). Corporate financial structure, misallocation and total factor productivity. *Journal of Banking & Finance, 39*, 177–191.
- Vives, X. (2008). Innovation and competitive pressure. *The Journal of Industrial Economics*, 56, 419–469.



Appendix. Proof of Proposition 1

We prove this in three cases, respectively.

Case 1. g(z, q) = z

From equation (6) and g(z, q) = z, if $\hat{z} < \underline{z}$, $(A - q)q - c(I)q - \frac{1}{2}I^2 - D$ is concave in q and I, and its maximum is achieved by its first-order optimal conditions.

If $z \leq \hat{z} < \overline{z}$, we have

$$V(q,I) = \int_{\hat{z}}^{\bar{z}} [(A-q)q - c(I)q + z - \frac{1}{2}I^2 - D]f(z)dz$$

$$= \frac{1}{\bar{z} - \underline{z}} \int_{\hat{z}}^{\bar{z}} zdz + [(A-q)q - c(I)q$$

$$- \frac{1}{2}I^2 - D] \frac{1}{\bar{z} - \underline{z}} \int_{\hat{z}}^{\bar{z}} dz$$

$$= \frac{1}{2(\bar{z} - \underline{z})} \{\bar{z}^2 - \hat{z}^2\} + \frac{1}{\bar{z} - \underline{z}} \hat{z}^2 + \frac{\bar{z}}{\bar{z} - \underline{z}} \hat{z}$$

$$= \frac{1}{2(\bar{z} - \underline{z})} (\bar{z}^2 + \hat{z}^2) - \frac{\bar{z}}{\bar{z} - \underline{z}} \hat{z}$$

$$= \frac{1}{2(\bar{z} - \underline{z})} (\bar{z} - \hat{z})^2.$$

(A1)

Apparently, if $\underline{z} \le \hat{z} < \overline{z}$, the above equation suggests that V(q, I) should attain its maximum at the point of the maximum $[(A - q)q - c(I)q - \frac{1}{2}I^2 - D]$. Therefore, equation (4) has the unique solution and conclusions are achieved for the first case.

Case 2. g(z, q) = zq

If g(z, q) = zq and $\hat{z} < \underline{z}$, obviously, the conclusion holds because the function $(A - q)q - c(I)q - \frac{1}{2}I^2 - D + zq$ is concave in q and I. Here $\underline{z} \le \hat{z} < \overline{z}$ is addressed. We have the following conclusions.

$$\begin{split} V(q,I) &= \int_{\frac{z}{q}}^{\overline{z}} \left[(A-q)q - c(I)q + zq - \frac{1}{2}I^2 - D \right] f(z) dz \\ &= \frac{1}{\overline{z} - \underline{z}} q \int_{\frac{z}{q}}^{\overline{z}} z dz + \frac{\hat{z}}{\overline{z} - \underline{z}} \int_{\frac{D-[(A-q)q - c(I)q - \frac{1}{2}^2]}{q}}^{\overline{z}} dz \\ &= \frac{q}{2(\overline{z} - \underline{z})} \{ \overline{z}^2 - \frac{1}{q^2} \hat{z}^2 \} + \frac{1}{q(\overline{z} - \underline{z})} \hat{z}^2 - \frac{\overline{z}}{\overline{z} - \underline{z}} \hat{z} \\ &= \frac{q}{2(\overline{z} - \underline{z})} \{ \overline{z}^2 + \frac{1}{q^2} \hat{z}^2 \} - \frac{\overline{z} \hat{z}}{\overline{z} - \underline{z}} \\ &= \frac{1}{2(\overline{z} - \underline{z})} \{ q^{\frac{1}{2}} \overline{z} - q^{-\frac{1}{2}} \hat{z} \}^2. \end{split}$$
(A2)

In the above equation, $q^{\frac{1}{2}}\overline{z} + q^{-\frac{1}{2}}[(A-q)q - c(I)q - \frac{1}{2}I^2 - D]$ is concave in q and I. V(q, I) should attain its maximum at the point of the maximum to $q^{\frac{1}{2}}\overline{z} + q^{-\frac{1}{2}}[(A-q)q - c(I)q - \frac{1}{2}I^2 - D]$. Equation (4), therefore, has the unique solution and conclusions are proved for the second case.

Case 3. g(z, q) = -zq

If g(z, q) = -zq and $\hat{z} < \underline{z}$, since the function $(A - q)q - c(I)q - \frac{1}{2}I^2 - D + zq$ is concave in q, apparently the conclusion holds.

Here $z < \hat{z} < \bar{z}$ is addressed. We have

$$V(q,I) = \int_{\frac{z}{q}}^{\overline{z}} [(A-q)q - c(I)q - zq - \frac{1}{2}I^2 - D]f(z)dz$$

$$= -\frac{q}{(\overline{z}-\underline{z})} \int_{-\frac{z}{q}}^{\overline{z}} zdz - \frac{\hat{z}}{\overline{z}-\underline{z}} \int_{-\frac{z}{q}}^{\overline{z}} dz$$

$$= \frac{-q}{2(\overline{z}-\underline{z})} \{\overline{z}^2 - \frac{1}{q^2} \hat{z}^2\} - \frac{1}{q(\overline{z}-\underline{z})} \hat{z}^2 - \frac{\overline{z}}{\overline{z}-\underline{z}} \hat{z}$$

$$= \frac{q}{2(\overline{z}-\underline{z})} \{-\overline{z}^2 - \frac{1}{q^2} \hat{z}^2\} - \frac{\overline{z}}{\overline{z}-\underline{z}}$$

$$= \frac{-1}{2(\overline{z}-\underline{z})} \{q^{\frac{1}{2}}\overline{z} - q^{-\frac{1}{2}}\hat{z}\}^2.$$
(A3)

Since the above equation is concave in q and I, V(q, I) should attain its unique maximum from first-order optimal conditions. Conclusions for the third case are also obtained.

The conclusions of this proposition are all achieved and the proof is complete. We further point out that it is impossible to obtain concave properties but the uniqueness of the solution is achieved. $\hfill \Box$



Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

